ComS 425: Homework # 1 — 60 points

Due January 23, 2007 in class

1. (5 points) If the performance of CPUs increases by a factor 10 every five years, how long does it take for performance to double?

2. The complexity of an algorithm often becomes more, not less, significant as computer speeds increase. For example, an aircraft designer runs one simulation each evening between 5 P.M. and 8 A.M. When she gains access to a faster computer, she uses this extra speed to run a larger simulation (one that can provide more detailed results) in the same amount of time. Suppose her current computer can solve a problem of size 100,00 in 15 hours. Assume that the execution time is determined solely by CPU speed; i.e., all other resources such as I/O bandwidth and primary memory are not a constraint on performance. How large a problem can be solved in 15 hours by a computer that is 100 times faster if the simulation program's time complexity is

   (a) (5 points) $\Theta(n)$

   (b) (5 points) $\Theta(n\ log_2\ n)$

   (c) (5 points) $\Theta(n^2)$

   (d) (5 points) $\Theta(n^3)$

3. (5 points) Name two advantages commodity clusters have over commercial parallel computers.
   Name one advantage a commercial parallel computer has over a commodity cluster.

4. (5 points) Prove that if node u is distance i from node v in a hypercube, then there are i paths of length i from u to v that share no edges.

5. (5 points) Prove that a hypercube has no cycles of odd length.

6. Consider the Omega network that connects p processors. Note that Omega network is a *blocking network*, i.e., if a pair of processors is communicating (via a set of 2×2 switch elements), another pair of processors wishing to communicate via the same (sub)set of switches may be prevented from doing so until the first pair finishes.

   Define a function f that maps $P = [0, 1, \ldots, p-1]$ onto a permutation P' of P, such that $P'[i] = f(P[i])$ and $P'[i] \in P$ for all $0 \leq i < p$. Think of this function as

mapping communication requests by the processors so that processor $P[i]$ requests communication with processor $P'[i]$.

(a) (5 points) How many distinct permutation functions exist?

(b) (5 points) How many of these functions result in non-blocking communication?

7. (10 points) Do some research and find, for each category in Flynn's taxonomy, at least one commercial computer fitting that category. (It is OK to name a computer that is no longer available, but you may not name a computer mentioned in the textbook or in class.)